Data-Informed Modeling of Milligram-Scale Quadrupedal Robot Locomotion

Walker Gosrich, Ryan St. Pierre, and Sarah Bergbreiter

***Abstract*— This work uses the microQuad, a quadrupedal, magnetically actuated robot, to examine legged locomotion at the milligram scale. microQuad locomotion data is recorded as the robot is actuated through a wide range of discrete frequencies between 5Hz and 50Hz, and average horizontal velocity and energy phasing information is extracted. The trends in these data, which indicate a passive gait shift in the higher portion of the frequency domain, are then compared to expected trends from classical mathematical models for biologically-inspired locomotion: theses models fail to predict a gait shift.Two new mathematical models are developed to better represent and explain locomotion at the milligram scale; these models effectively represent locomotion in the low frequency domain, and can help to conceptually explain the gait shift occurring at higher frequencies.**

I. INTRODUCTION

Biological locomotion at the milligram scale is highly effective. Insects such as cockroaches, ants, and mites run efficiently and robustly at high speeds ranging from around 10 bodylengths per second for cockroaches and ants to hundreds of bodylengths per second in certain species of mite [1]. However, the mechanics of this motion are not well understood.

Simple mathematical models of legged locomotion such as the spring loaded inverted pendulum (SLIP) running model [2] and the inverted pendulum (IP) walking model [3] have been used to help understand and characterize locomotion in scales greater than one gram, such as animals of varying sizes and gaits [3], humans [4], cockroaches [5][6], and ants [7]. They have also been applied to robots in the kilogram scale [8][9] and the gram scale [10] to create control schema and help to understand locomotion. However, these models are as yet untested at the milligram scale of the microQuad, where dynamic motion and ground contact effects make locomotion difficult to model.

In order to achieve locomotion as effective as insects at the milligram scale, the underlying mechanics must be well understood. In this work, the microQuad will be used as a physical model to aid in evaluating existing models and developing new models for small-scale locomotion to work towards this understanding.

II. ROBOT SYSTEM AND EXPERIMENTAL SETUP

The microQuad (Figure 1) is a microrobot that weighs 1mg, measures 2.5mm in length, and has a leg length of 705μm. At this weight, it is the smallest terrestrial legged robot. It is magnetically actuated, with torque applied by an external magnetic field to a 250μm cubic neodymium magnet in each leg. The microQuad is 3D printed fully assembled – with simple 1 degree of freedom pivot joints in each hip – by the Nanoscribe 3D printer. After magnets are glued into the hip sockets, the robot is complete.

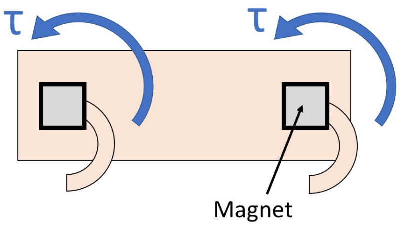

Fig. 1. The microQuad: the smallest and fastest terrestrial legged robot. Torque applied in the direction of rotation on the magnets is indicated with blue arrows.

The experimental setup (Figure 2) consists of the microQuad, a two-axis Helmholtz coil, an acrylic slide, a Tektronix AFG3022C function generator, a FASTCAM Mini UX100 high speed camera, a light source, and a macro camera lens. The slide, with attached legs, is first placed in the two-axis Helmholtz coil, and marked at the boundary of the zero-gradient area of the generated magnetic field. The slide is removed, and the microQuad is positioned on one of the markings, such that it will run directly through the zero-gradient field area. The slide is reinserted into the Helmholtz coil, and the camera positioned to capture the full zero-gradient field area.

To capture data, the function generator is powered on: one output is connected to each axis of the coil, and energized by a sinusoidal signal of +/- 5V from the function generator. These signals are aligned to be 90°out of phase, such that a two-dimensional rotating magnetic field is generated within the Helmholtz coil at the sinusoidal signal frequency.

Once video data has been recorded, the robot is tracked throughout the trial at three points: the top front corner of the chassis, the top rear corner of the chassis, and the tip of the leg (toe) that is most representative of the four legs throughout the trial. This tracking is performed using TEMA Automotive marker-less tracking software.

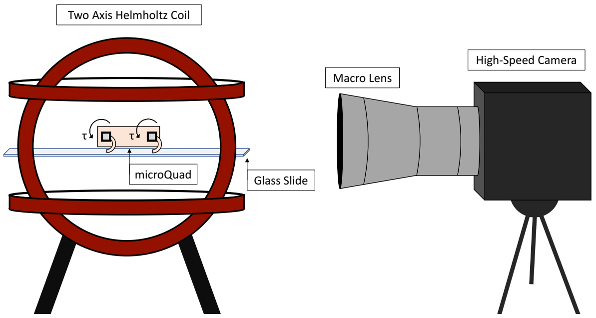

Fig. 2. The test setup, showing the Helmholtz Coil with the microQuad resting on a slide in the center, and the high speed camera equipped with macro lens on the right of the frame

III. MOTION CHARACTERIZATION METRICS

Two primary metrics were used to characterize the microQuad's mode of locomotion throughout a trial. These metrics – average horizontal velocity, and energy phasing – capture the overall effectiveness of the robot's locomotion, and indicate properties of efficiency and energy transfer.

These two metrics are also used to compare models against the experimental locomotion data. In testing the fit of a model, the average horizontal velocity, and its change as frequency is increased, indicates the accuracy of the model in terms of scaling and macroscopic behavior. The energy phasing characterizes the nature of the locomotion in each cycle. Out-of-phase energy profiles are characteristic of stiff-legged walking gaits, and in-phase energy profiles are characteristic of springy-legged gaits akin to running in humans and other large-scale animals.

A. Average Velocity

The average velocity represents a scaling metric, and an overall metric of the robot's performance over each trial. The average velocity is calculated simply as the horizontal displacement of the robot divided by total time in a trial. This metric was calculated for each trial over the frequency sweep, and offers further information when compared with frequency: its first derivative implies whether the gait at a certain frequency is dominated by kinetic or dynamic effects. If the gait is dominated by kinetic effects, the first derivative will be a constant, scalar multiple of the stride length. However, if the gait is dominated by dynamic effects, the first derivative will not be a function of stride length: instead it will depend on other factors.

B. Energy Phasing

The energy phasing is the time difference between peaks in the curves of horizontal kinetic energy (HKE) and gravitational potential energy (GPE), as a proportion of the overall cycle time. A 0°phase shift indicates that the signals are entirely in phase, whereas a 180°phase shift indicates the signals are fully out of phase. Additionally, a negative phase shift indicates that the HKE peaks occur before the GPE peaks.

IV. MODELS

Two models were chosen to characterize the motion of the microQuad. Both were based upon the IP model introduced in [3], consisting of a point mass atop a rigid leg.

A. Grounded Model

The grounded model represents locomotion in two phases: an active vaulting phase, in which the point mass vaults over the leg under a constant angular velocity prescribed by the actuation frequency of the system, and a passive leg swing phase, in which the point mass remains motionless as the massless leg rotates at the same angular velocity (Figure 3). In this passive rest phase, the point mass sits at a y-position defined by the robot's body height: in this case, it is the y-location of the center of mass of the microQuad while resting on the ground: $343.75\mu m$. In the grounded model, there is no force analysis. It is assumed that enough torque is supplied by the magnetic field to maintain a constant angular velocity, and that ground contact is "sticky": there are no slipping considerations, and negative ground reaction forces are allowed.

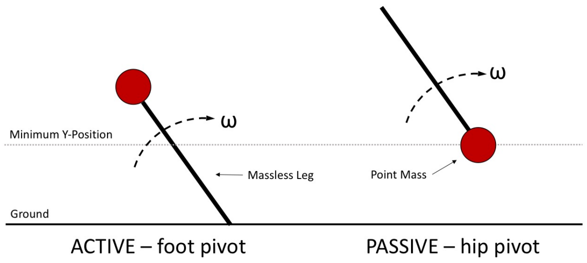

Fig. 3. Grounded model depiction. In the active vaulting phase, the point mass rotates about the tip of the leg. In the passive resting phase, the massless leg rotates about the point mass.

B. Model with Aerial Phase

In this model, assumptions are changed to allow an aerial phase. It is still assumed that a constant angular velocity is maintained, but negative ground reaction forces are not permitted: if the centripetal acceleration necessary to maintain constant rotation about the leg tip is not supplied by gravity, the point mass launches into ballistic motion, governed by the velocity vector of the point mass upon takeoff. The flight phase ends by landing on either the point mass, and transitioning directly into the rest phase described in the previous section, or by landing on the foot, and transitioning immediately into the vaulting phase.

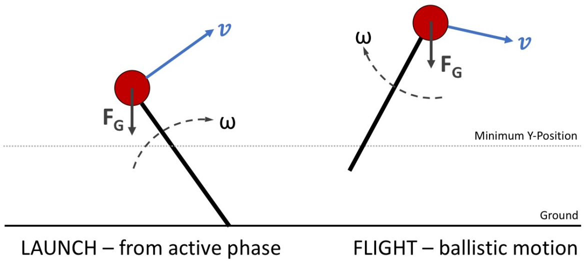

Fig. 4. Depiction of the model with aerial phase. If centripetal acceleration in the y-direction exceeds acceleration due to gravity, the point mass launches (left) into the flight phase (right). In this phase, ballistic motion is governed by the velocity vector from the launch, shown in blue.

V. RESULTS

The microQuad was actuated at 12 distinct frequencies between 5Hz and 50Hz.

From the trajectories of the top front, top rear, and toe throughout the trial, the position of the center of gravity, the pitch of the robot chassis, and the angular position of the toe are calculated at each time interval. These data are then used to calculate the horizontal and vertical velocities throughout the trial, and the angular velocities of the toe throughout the trial. Y-axis position of the toe is used to determine ground contact.

Both the average velocity and phase shift trends of the experimental data show that the microQuad experiences a shift in locomotion characteristics through the frequency space. At frequencies below 30Hz, the microQuad exhibited almost entirely kinematic motion, with average forward velocity changing linearly with frequency, and almost completely in-phase GPE and HKE profiles. At higher frequencies, the locomotion began to exhibit more dynamic behaviors, with average forward velocity peaking at 35Hz and then decreasing, and the phasing of GPE and HKE shifting to a maximum of 150°out of phase (with HKE leading) at 50Hz.

A. Average Horizontal Velocity Characterization

As seen in Figure 5, the average horizontal velocity of the microQuad is highly linear at actuation frequencies from 5Hz to 25Hz. This is due to the kinematic nature of the locomotion up to this point. The slope of this linear phase of the average velocity curve is roughly 0.35BL, or 0.875mm. This is remarkably near the stride length of the microQuad in its kinematic motion: 0.906mm. This indicates that the motion in this portion of the frequency space is entirely governed by the contact kinematics of the robot: as frequency increases, horizontal velocity increases linearly with the additional number of strides taken per second - an additional cycle per second results in an additional stride per second.

This highly kinematic behavior is modeled with high accuracy by both the grounded model and by the model with an aerial phase. This similarity between the models is present because there are not sufficient velocities to launch into the aerial phase in the low-frequency domain. As seen in Figure 5, the horizontal velocity vs. frequency trend matches almost exactly for the models and the experimental data.

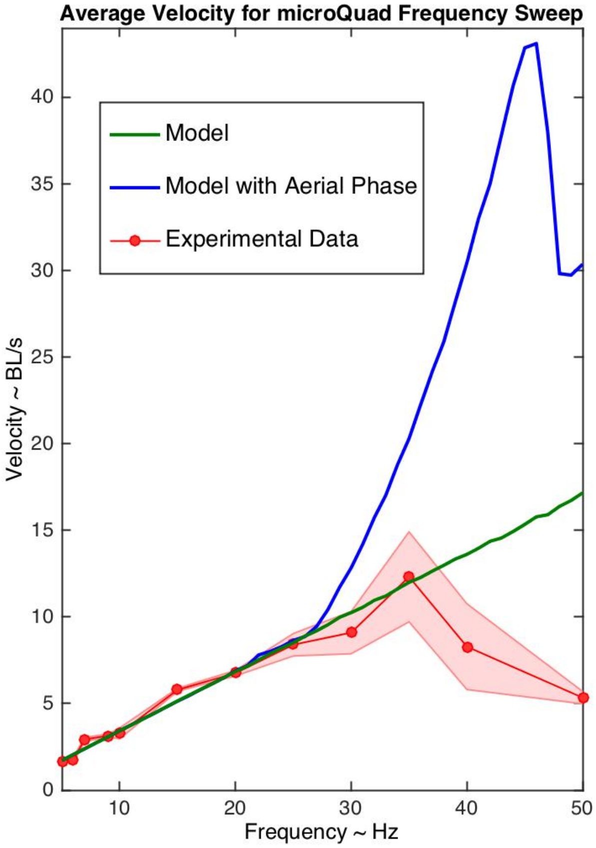

Fig. 5. The average horizontal velocity, in bodylengths per second, of the microQuad as it is actuated at different frequencies. All averages were taken across the entire trial, and then all three trials were averaged together for each frequency.

B. Energy Phasing Characterization

As seen in Figure 6, the HKE and GPE are close to in phase for the portion of the frequency domain under 30Hz. There is some oscillation about the 0°mark, but the maximum amount of this difference is roughly 50°, a small phase shift in the context of the higher frequencies.

Both of the models also exhibit zero phase shift in the low frequency domain. The grounded model remains completely in phase through all frequencies: because there are no dynamic effects, there is no reason to depart from this SLIP-like energy trend.

At higher frequencies, however, the experimental data depart from the line of zero phase shift. The experimental data show that the microQuad begins to exhibit an energy phasing shift at 35Hz – the actuation frequency at which it ran fastest – of about -60°. As actuation frequency increases, the phase shift also increases in a linear trend ending with a maximum of about -150°of phase shift at 50Hz actuation frequency

The model with an aerial phase also shows this departure from the line of zero phase shift, albeit beginning earlier and with a greater slope. This model begins to experience phase shift at around 18Hz actuation frequency, which decreases as actuation frequency increases, to -350°– nearly a full cycle – at the 50Hz actuation frequency.

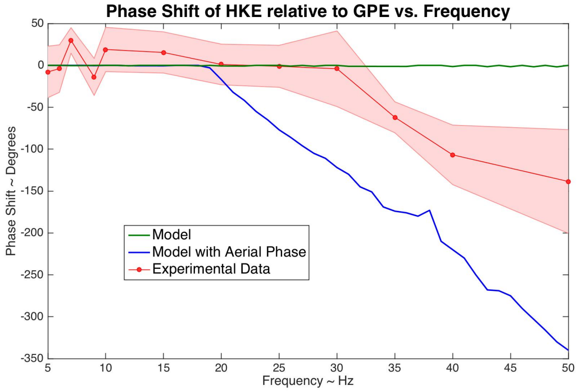

Fig. 6. The phase shifts of HKE relative to GPE vs. actuation frequency

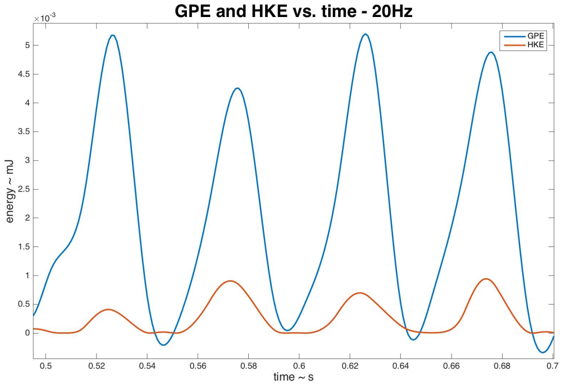

Fig. 7. The horizontal kinetic energy (HKE) and gravitational potential energy (GPE) of the microQuad vs. time across a 20 Hz trial

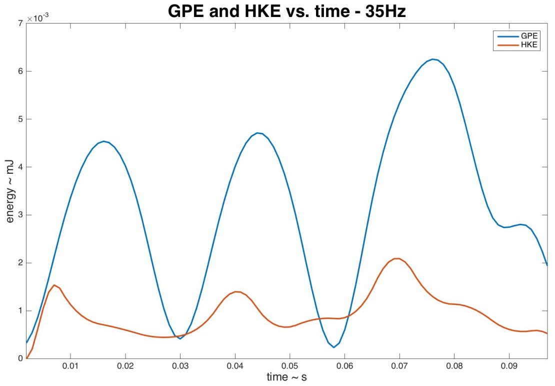

Fig. 8. The horizontal kinetic energy (HKE) and gravitational potential energy (GPE) of the microQuad vs. time across the 35 Hz trials, constructed the same way as the 20Hz trials

VI. DISCUSSION

A. Low-Frequency Actuation Domain

Both models were highly successful in representing the experimental data in the low-frequency domain, where average horizontal velocity increases linearly corresponding to stride length, and there are only small quantities of energy phase shift. This predictability of the microQuad locomotion is likely due to the highly kinematic nature of movement at the lower frequencies. Because aerial phases of microQuad running are short and have little effect in the low frequency domain, the movement is dominated by the simple vaulting motion of the robot's center of gravity over the legs, and is easily characterized by the stride length.

Examination of the videos of the microQuad running show that ground contact, though not perfect, is of sufficient quality that it can be neglected in a model that still accurately predicts motion.

B. High-Frequency Actuation Domain

Average Velocity

In the high-frequency actuation domain, above 25Hz, the microQuad gait is less easily characterized. Firstly, the average velocity of the microQuad increases to a peak at 35Hz and then suffers from a performance decrease. The model with aerial phase can help to explain this trend, but cannot sufficiently represent the microQuad gait through this actuation domain.

The model with aerial phase shows – and video examination confirms – that the average velocity vs. frequency peak occurs when one full rotation of the legs occurs while the robot is in the air, and the robot lands on its legs. In this mode, the rest phase that was present at lower frequencies is practically non-existent, and the robot begins the next stride immediately after the legs contact the ground.

As actuation frequency is increased beyond this peak, the robot begins to perform more than one full leg rotation during its aerial phase. This means that the robot falls onto its body, directly into the rest phase, and then must complete another full leg rotation while resting before beginning the next stride.

Although the model does help explain this drop in performance following the peak, it drastically overpredicts the robot's performance as soon as the aerial phase begins to occur, and mispredicts the location of the peak. It is hypothesized that this disparity is due to two factors: the primary factor is the ground contact of the microQuad at higher frequencies. It is apparent in the videos of microQuad running that there is significant slip occurring between the legs and the acrylic slide, especially as the legs move from first contact with the ground to being positioned directly under the robot. This slip causes a large portion of the energy that would be transferred to horizontal velocity to instead be transferred to vertical velocity. This results in less efficient movement, and also means that the robot jumps higher and has a longer aerial phase than would be predicted. This longer aerial phase increases the amount of leg rotation that occurs during the aerial phase, and decreases the actuation frequency required to complete a full rotation during the aerial phase.

The secondary factor is the limited torque available to the microQuad. Because an assumption of constant angular velocity is made for the model, torque is permitted to far exceed the quantities that are provided by the actuation system in the launch phase. This results in an aerial phase that is far too powerful, and causes the robot to travel too high and too far.

Energy Phasing

The energy phasing of the HKE vs. GPE also becomes more difficult to model as the actuation frequency increases above 25Hz. As seen in Figure 6, the phase shift drops below the zero line at 30Hz, and decreases linearly from there, to a maximum of about -150°– about half of a cycle of phase shift. It is hypothesized that this shift from completely in phase to almost completely out of phase is due, again, to ground contact and dynamic factors. It is evidenced by the video data that part of the reason for the decreasing phase shift is the movement of the maximum kinetic energy from the peak of the vaulting phase to the beginning of the vaulting phase. When there is an aerial phase, and poor ground contact is occurring, the microQuad propels itself forward rapidly for the first portion of the leg contact, and then begins to travel upwards, losing horizontal speed, as the leg slips to complete its rotation. As frequency increases and the aerial phase increases in duration, encompassing more than one full rotation, the peak in HKE becomes further separated from the peak of GPE.

An additional factor that is observed in the microQuad locomotion at high frequencies is the maintenance of HKE throughout the gait cycle. At lower actuation frequencies, the microQuad came to a complete halt between each gait cycle, as can be seen in the return of HKE to 0 between each stride in Figure 7. As actuation frequencies exceeded 30Hz, the minimum HKE increased from zero, as evidenced by the maintenance of about $0.5\mu J$ of HKE throughout the three strides depicted in Figure 8.

Again, the model with aerial phase helps to explain the above noted behavior, as seen by its continuous horizontal velocity, and by its trend of phase shift vs. actuation frequency: departing from the zero line and decreasing. However, the model once again overpredicts the amount of phase shift. Similarly to the average horizontal velocity, this is due to the assumption of perfect ground contact, and the impossible torque required by the assumption of constant angular velocity.

The IP model without the aerial phase does not exhibit any phase shift, because it maintains ground contact throughout the gait cycle. It is not an accurate model for the microQuad's locomotion in the high frequency domain.

VII. CONCLUSIONS

The microQuad is the fastest moving and smallest terrestrial legged robot, weighing roughly one milligram, and moving at a maximum of 14.8 BL/s. Over the range of actuation frequencies examined in this study – 5Hz to 50Hz – it exhibited simple kinematic walking below 30Hz, which was easily characterized by models, and exhibited more complex dynamic running at higher frequencies, which was less effectively represented by the models.

In order to fully understand locomotion at these speeds and at this size scale, the ground contact effects must be measured, by detecting the ground reaction forces over a microQuad running trial. Once these can be obtained, locomotion can be thoroughly and accurately modeled.

Until this measurement is possible, the models presented in this work are a valuable aid in the understanding of microQuad locomotion, and suggest that in the future, steps must be taken to maintain kinematic running in a grounded phase over higher frequency domains to achieve highly effective locomotion comparable to the microQuad's fastest counterparts.

REFERENCES

[1] S. Rubin, J. Wright, M. Young, D. Whitaker, and A. Ahn, "Exceptional locomotory performance in Paratarsotomus macropalpis mites," *The FASEB Journal*, vol. 28, p. Supplement 878, 4 2014.

[2] R. Blickhan and R. Full, "Similarity in multilegged locomotion: Bouncing like a monopede," *Hournal of Comparative Physiology A*, no. 173, pp. 509–517, 1993.

[3] G. A. Cavagna, N. C. Heglund, and C. R. Taylor, "Mechanical work in terrestrial locomotion: two basic mechanisms for minimizing energy expenditure," *American Journal of Physiology - Regulatory, Integrative and Comparative Physiology*, vol. 233, pp. R243–R261, 1977.

[4] G. Cavagna, H. Thys, and Zamboni, "The sources of external work in level walking and running," *The Journal of physiology*, vol. 262, no. 3, pp. 639–657, 1976.

[5] R. Full and M. Tu, "Mechanics of Six-Legged Runners," 1990.

[6] J. Schmitt and P. Holmes, "Mechanical models for insect locomotion: dynamics and stability in the horizontal plane I. Theory," *Biological Cybernetics*, vol. 83, no. 6, pp. 501–515, 2000.

[7] L. Reinhardt and R. Blickhan, "Level locomotion in wood ants: evidence for grounded running," *Journal of Experimental Biology*, vol. 217, no. 13, pp. 2358–2370, 2014.

[8] C. Pang, M. K. Kwak, C. Lee, H. E. Jeong, W. G. Bae, and K. Y. Suh, "Nano meets beetles from wing to tiptoe: Versatile tools for smart and reversible adhesions," *Nano Today*, vol. 7, no. 6, pp. 496–513, 2012.

[9] S. Soyguder and H. Alli, "Kinematic and dynamic analysis of a hexapod walking-running-bounding gaits robot and control actions," *Computers and Electrical Engineering*, vol. 38, no. 2, pp. 444–458, 2012.

[10] B. D. Miller, J. E. Clark, B. Goldberg, N. Doshi, K. Jayaram, and R. J. Wood, "Gait studies for a quadrupedal microrobot reveal contrasting running templates in two frequency regimes,"